

P1.1

There is a drawer in Katie's house which contains twelve ribbons – four red, four white and four blue. Her mother sent her into the room for two ribbons of the same colour, but it didn't matter which colour. However, it was very dark and there were no candles, torches or any other source of light.

How many ribbons would Katie have to bring out to make sure that she had two ribbons of the same colour?

Solution

If you took three ribbons you might get, for example, RWB. However, if you took four ribbons then you must have two of the same colour.

Consider the following in which there is at least one R.

RRRR

RRRW

RRWW

RWWW

RRRB

RRWB

RWWB

RRBB

RWBB

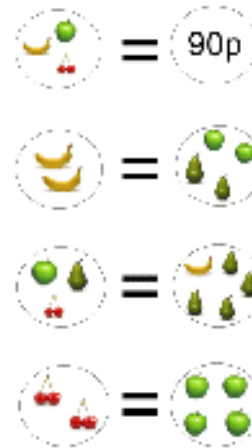
RBBB

And similarly if the list started with four blues or with four whites.

P1.2

A mathematical greengrocer had the display shown alongside in his shop window. The poster shows how the costs of one apple, one banana, one pear and a pair of cherries are related.

How much does each kind of fruit cost?



Solution

1. Non-algebraic

From picture 4, a pair of cherries cost the same as 2 apples.

Using this with picture 3, 3 apples and a pear cost the same as a banana and 4 pears. So 3 apples cost the same as a banana as 3 pears,

But from picture 2, a banana costs the same as an apple and a pear so the previous statement becomes 3 apples cost the same as an apple and 4 pears or 2 apples costs the same as 4 pears so 1 apple costs the same as 2 pears.

So a pair of cherries costs the same as 4 pears and a banana costs the same as 3 pears.

We can now interpret picture 1 and obtain the cost of 9 pears is 90p and hence 1 pear costs 10p, a pair of cherries cost 40p, a banana costs 30p and an apple costs 20p.

2. Algebraic

$$b + c + a = 90 \quad (1)$$

$$b = a + p \quad (2)$$

$$a + c = b + 3p \quad (3)$$

$$c = 2a \quad (4)$$

Eliminating c

$$b + 3a = 90 \quad (5)$$

$$b = a + p \quad (6)$$

$$3a = b + 3p \quad (7)$$

Eliminating b

$$4a + p = 90$$

$$2a = 4p \quad (9)$$

Hence

$$9p = 90$$

So the costs are:

Pear = 10p, Apple = 20p, Banana = 30p, A pair of Cherries = 40p

P1.3

Alice, Bill, Colin, Daisy and Edward play a game in which each is a frog or a toad. A frog's statements are always false and a toad's statements are always true.

- | | | | |
|---|--|---|-------------------------------------|
| A | Alice says that Bill is a toad. | B | Colin says that Daisy is a frog. |
| C | Edward says that Alice is not a frog. | D | Bill says that Colin is not a toad. |
| E | Daisy says that Edward and Alice are different kinds of animals. | | |

Who of Alice, Bill, Colin, Daisy and Edward are the frogs?

Solution

Label the statements A - E.

First, assume Alice is a toad. This means Bill is a toad.

And hence from D, Colin is a frog.

So B is not true, so Daisy is a toad.

Thus E is true, and, since Alice is a toad, Edward is a frog.

Finally C is false, so Alice is a frog.

But this contradicts the original assumption that Alice is a toad which must therefore be wrong.

So actually Alice is a frog. Then A gives that Bill is a frog.

And hence D is false, so Colin is a toad.

Then B gives Daisy is a frog.

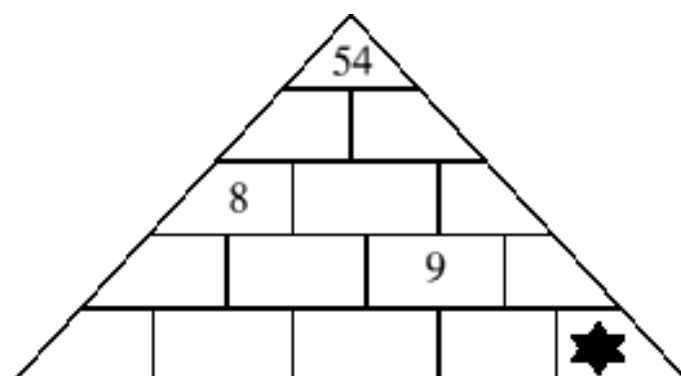
And E is false, so Edward is a frog.

Finally C is false, so Alice is a frog.

This matches with the initial statement that Alice is a frog, and so we have the solution.

Alice, Bill, Daisy and Edward are all frogs.

P1.3



In each region of the triangle shown there is a whole number, three of which are given. Each number is the sum of the two numbers immediately below it and all numbers are different. Find out which number must be in the region marked with the star and explain why.

Solution

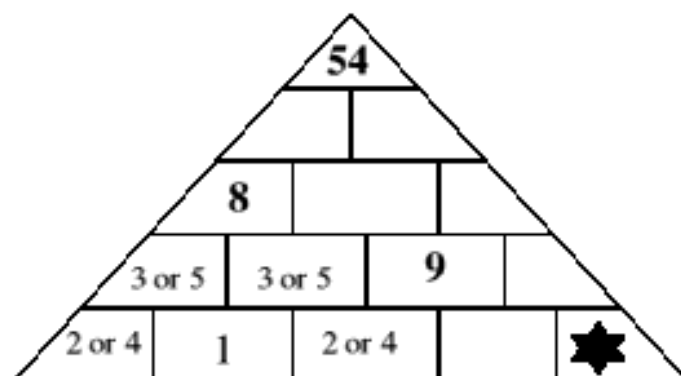
As the numbers are all different, the smallest possible numbers in the bottom row are 1, 2, 3, 4 and 5. Thus the smallest possible number in the second row is 3.

To get the 8, the cells below it must contain 3 and 5.

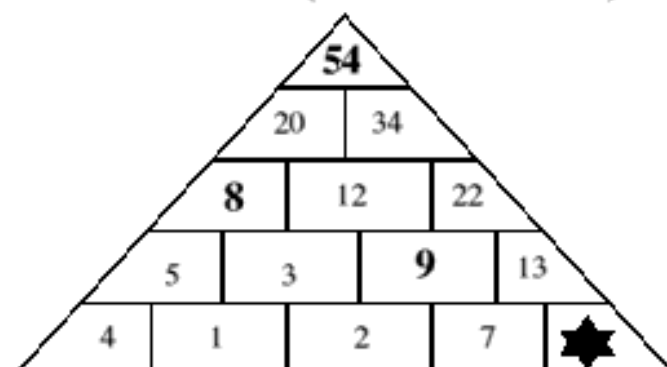
Below the 3, we must have 1 and 2.

Below the 5, we could have 2 and 3 or 1 and 4, but 3 cannot be repeated so 5 comes from $1 + 4$.

As 1 feeds into 3 and into 5, it must be directly below the 8 with 4 and 2 on either side. So we have:



But 4 cannot feed into 9 as 5 is already used. So 4 is on the left. This means that below 8 we have 5 and 3 (rather than 3 and 5).



The rest can now be filled in: 12, 20, 34, 13, 7 and finally the required number is 6.